Using an FFT to Evaluate Periodic Signals

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Abstract *This paper discusses the concept of a fast Fourier transform and how it can be used for analyzing periodic signals. The algorithm itself is fairly simple and can be implemented using Microsoft Excel. However, the selection of sampling parameters is not intuitive. This paper will describe how to select appropriate parameters, and how to verify that the algorithm results are accurate. The code samples included in this document are written in Excel VBA.*

1 Introduction

The FFT is an algorithm which can convert a signal in the time domain into a discrete frequency domain. The immediate practical value of this is analysis of periodic signals, such as vibration signals, electrical harmonics, and induced signal noise. Knowing the frequencies at which a signal is occurring is very valuable in troubleshooting. Given that an accurate sampling of a periodic signal can be obtained, the FFT can be used to break down a periodic signal into individual frequency components.

2 The Equation

The FFT equation is as follows:

$$X_k = \sum_{n=0}^{N-1} x[n] e^{-j*2*\pi*n*k/N} \tag{1}$$

Where X_k is a complex value corresponding to the kth frequency. N is the number of samples collected. And $x[n]$ are the samples, which are real numbers. Each value of X_k has a real part and an imaginary part. The magnitude of X_k is related to the magnitude of the signal at the kth frequency. The angle of X_k is related to the phase shift of the signal at the kth frequency.

3 Example Data Set

A sample data set was created in Excel using $y(t) = 1 + 2cos(10t + \frac{\pi}{4}) + 3cos(20t - \frac{\pi}{3}) + 4cos(30t)$. This discrete array uses a time vector of 102 Hz, or a time interval of $\frac{1}{102}$ seconds. There are 128 samples in the set. The sampling rate was chosen in order to maximize the periodicity of this signal and to select integer values for frequency bins. Note that the signal is not perfectly periodic. Table 1 shows how many periods are present in the signal for each frequency. These are very close to an integer number, which would make them perfectly periodic. However even the small error will show up in the data. In this case the error will be small enough to ignore.

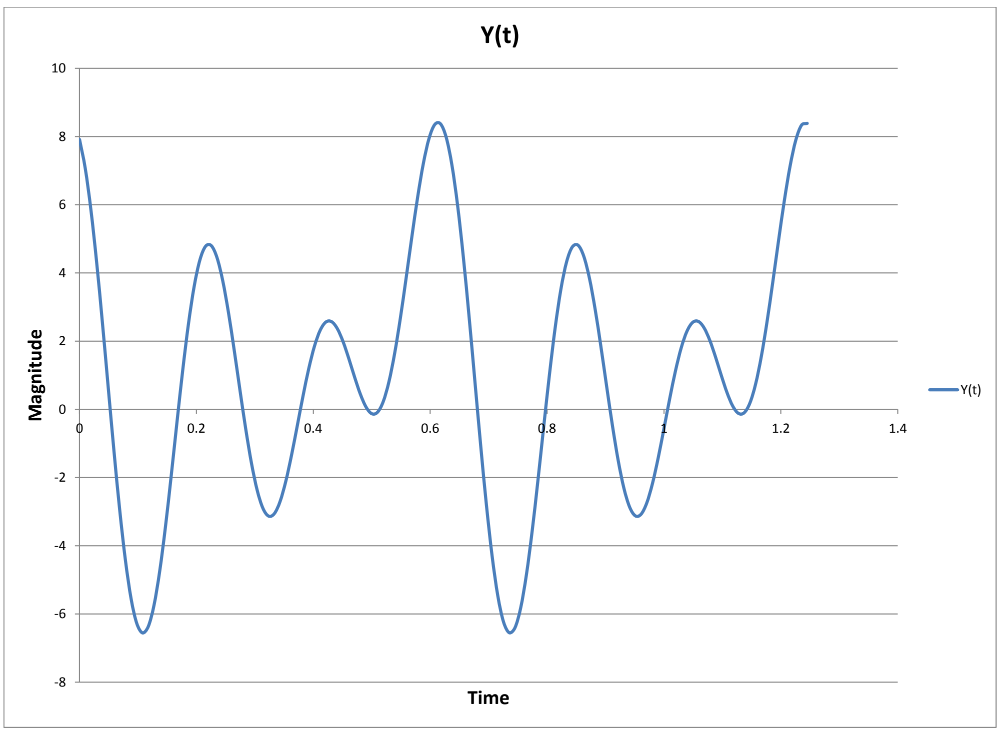

Figure 1: $y(t) = 1 + 2cos(10t + \frac{\pi}{4}) + 3cos(20t - \frac{\pi}{3}) + 4cos(30t)$