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INFORMATION

If we want to find the probability that a given number of recurrences of an event occurs in a fixed time interval, and we know the average rate of the occurrence of the event and that each occurrence happens independently of the time the last event occurred, then we can use the Poisson probability distribution.

Examples of things we can model with a Poisson distribution:

- the number of soldiers killed by horse kicks each year in each corps of the Prussian cavalry (famous example from a book by Ladislaus Josephovich Bortkiewicz (1868-1931).)
- the number of phone calls arriving at a call center per minute
- number of goals in sports involving two competing teams
- number of deaths per year in a given age group
- number of jumps in a stock price in a given time interval
- number of mutations in a given stretch of DNA after a certain amount of radiation

This part is from our text:

Think of the time period as being split up into n subintervals, each of which is so small that at most one event could occur in it with probability greater than zero.

Let p be the probability that an event occurs in any subinterval. Then, for all practical purposes, we have $P(\text{no events occur in a subinterval}) = 1-p$

$P(\text{one event occurs in a subinterval}) = p$

$P(\text{more than one event occurs in a subinterval}) = 0$.

Then the total number of events in the time period is just the total number of subintervals that contain one event. If the occurrence of events can be regarded as independent from interval to interval, then the total number of events has a binomial distribution.

There isn't a unique way to choose the subintervals, so we know neither n nor p , so it seems reasonable that as we divide the time period into a greater number n subintervals, the probability p of one event in one of these shorter subintervals will decrease.

Let $\lambda = np$ and take the limit of the binomial probability $p(y) = nyp^y(1-p)^{n-y}$ as $n \rightarrow \infty$, we have $\lim_{n \rightarrow \infty} nyp^y(1-p)^{n-y} = \lim_{n \rightarrow \infty} \frac{n(n-1)\cdots(n-y+1)}{y!} \left(\frac{\lambda}{n}\right)^y \left(1 - \frac{\lambda}{n}\right)^{n-y}$
 $= \lim_{n \rightarrow \infty} \frac{\lambda^y}{y!} \left(1 - \frac{\lambda}{n}\right)^n \frac{n(n-1)\cdots(n-y+1)}{n^y} \left(1 - \frac{\lambda}{n}\right)^{-y}$
 $= \frac{\lambda^y}{y!} \lim_{n \rightarrow \infty} \left(1 - \frac{\lambda}{n}\right)^n \left(1 - \frac{\lambda}{n}\right)^{-y} \left(1 - \frac{1}{n}\right) \left(1 - \frac{2}{n}\right) \cdots \left(1 - \frac{y-1}{n}\right).$

Recall that $\lim_{n \rightarrow \infty} \left(1 - \frac{\lambda}{n}\right)^n = e^{-\lambda}$.

and note that all the other terms to the right of the limit have a limit of 1, so we get $p(y) = \frac{\lambda^y}{y!e^{-\lambda}}$.

A random variable with this probability function is said to have a *Poisson* distribution. Hence, Y , the number of events per week, has the Poisson distribution just derived.

[Exercise 3.1.22] Customers arrive at a checkout counter in a department store according to a Poisson distribution at an average of seven per hour. During a given hour, what are the probabilities that

1. no more than three customers arrive?

Answer: Let Y be the number of customers that arrive during the hour. Then Y is Poisson with $\lambda = 7$. So we have $P(Y \leq 3) = .0818$

2. at least two customers arrive?

Answer: $P(Y \geq 2) = .9927$

3. exactly five customers arrive?

Answer: $P(Y = 5) = .1277$.