

Chapter 1

First chapter

Definition 1.1. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is *continuous* in a point x_0 if

$$\lim_{x \rightarrow x_0} f(x) = f(x_0).$$

Definition 1.2. A función $f : \mathbb{R} \rightarrow \mathbb{R}$ is *derivable* in a point x_0 if

$$\lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

exists and is finite. In such a case, this limit is called the *derivative* of f in x_0 and is denoted by $f'(x_0)$.

The following result states the fundamental relation existing between the notions introduced in Definitions 1.1 and 1.2.

Theorem 1.1. *If $f : \mathbb{R} \rightarrow \mathbb{R}$ is derivable in x_0 , then f is continuous in x_0 .*

Chapter 2

Second chapter

Theorem 2.1 (Rolle's Theorem). *Let $f : [a, b] \rightarrow \mathbb{R}$ be a function continuous on $[a, b]$, derivable on (a, b) , and such that $f(a) = f(b)$. Then, there exists $c \in (a, b)$ such that $f'(c) = 0$.*

Theorem 2.2 (Mean Value Theorem). *Let $f : [a, b] \rightarrow \mathbb{R}$ be a function continuous on $[a, b]$ and derivable on (a, b) . Then, there exists $c \in (a, b)$ such that $f(b) - f(a) = f'(c)(b - a)$.*

Corollary 2.3. *Let $f : [a, b] \rightarrow \mathbb{R}$ be a function continuous on $[a, b]$ and derivable on (a, b) . If, for all $x \in (a, b)$, $f'(x) \geq 0$, then f is increasing in $[a, b]$.*

Theorem 2.4 (Cauchy's Mean Value Theorem). *Let $f, g : [a, b] \rightarrow \mathbb{R}$ be two functions continuous on $[a, b]$ and derivable on (a, b) . Then, there exists $c \in (a, b)$ such that $g'(b)(f(b) - f(a)) = f'(c)(g(b) - g(a))$.*

Main definitions

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Main results

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