Praxis 1: A study Guide

Math 1

Math 2

Chapter 1

Fractions

A general fraction has the form $\frac{a}{b}$, where $b \neq 0$. The variable a is the numerator and b is the denominator. This fraction can also be considered "a parts out of b parts", or "a divided by b". The denominator specifies the number of parts in a whole, while the numerator tells how many parts you have out of the whole. An example of a fraction would be $\frac{3}{4}$. Here $a = 3$ and $b = 4$. This tells you,"3 parts out of 4." An *improper fraction* such as $\frac{7}{3}$ is one in which the numerator is greater than the denominator, and indicates a fraction value greater than 1. A *mixed number* contains both a whole number and a fraction, such as $3\frac{2}{3}$.

Simplifying Fractions

When simplifying fractions we want to make sure that the fraction is **always** in its lowest terms. To write a fraction in lowest terms, we have to find the largest common factor. Take a look at Example 1 below. To simplify $\frac{12}{27}$, we must find the largest common factor of 12 and 27 and cancel that factor out (a number divided by itself equals 1). Continue to do this until you have found the simplest form. It is important to try to find the largest factor of both numerator and denominator. We see that the largest common factor of 12 and 27 is 3.

Example 1 Simplify $\dfrac{12}{27}$.

$$\frac{12}{27} = \frac{4 \cdot 3}{9 \cdot 3} = \frac{4 \cdot \cancel{3}}{9 \cdot \cancel{3}} = \frac{4}{9}$$

Example 2 Simplify $\dfrac{11}{121}$.
In Example 2, 11 and 121 share a common factor of 11, since both 121 and 11 are divisible by 11. Rewrite 121 and 11 as factors of 11, and *cancel* the common factor to find the simplified fraction.

$$\frac{11}{121} = \frac{1 \cdot 11}{11 \cdot 11} = \frac{1 \cdot \cancel{11}}{11 \cdot \cancel{11}} = \frac{1}{11}$$

Mixed Number to an Improper Fraction

Example 3 Convert $3\dfrac{2}{5}$ to an improper fraction.
To convert a mixed number to an improper fraction, multiply the whole number by the denominator, and add the result (product) to the numerator. Then divide the result by the same denominator. For this example, multiply 3 (the whole number) by 5 (the denominator) to find the product of 15. Add 15 to the numerator of the fraction to find the numerator of the improper fraction ($15 + 2 = 17$). The denominator does not change. The reason the denominator stays the same can be explained by the breakdown of $3\frac{2}{5}$ $3\frac{2}{5} = \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{2}{5}$, notice, the denominator is all fifths, we are just simplifying the mixed number to make an improper fraction.

$$\begin{aligned} 3\frac{2}{5} &= \frac{(3 \cdot 5 + 2)}{5} \\ &= \frac{17}{5} \end{aligned}$$

Example 4 Convert $2\dfrac{6}{7}$ to an improper fraction.
In Example 4, the product of 2 and 7 (14) is added to 6 ($14 + 6 = 20$) to find the numerator of the improper fraction.

$$
\begin{aligned}
2\frac{6}{7} &= \frac{(2\cdot 7)+6}{7} \\
&= \frac{20}{7}
\end{aligned}
$$

Improper Fraction to a Mixed Number

In the examples above, we started with a mixed number and converted to an improper fraction. In this case, we start with an improper fraction and convert to a mixed number. To simplify, we see how many times the denominator goes into the numerator evenly. Then what is left becomes the fraction. The denominator will stay the same.

Example 5 Write $\frac{23}{7}$ as a mixed number.

To convert an improper fraction to a mixed number, divide the numerator by the denominator. We see that 7 does not go into 23 evenly, but 7 goes into 21 three times with 2 left over. Put the remainder of the division operation over the original denominator to find the fraction portion of the mixed number. So, 3 is our whole number (out front) the remainder is 2 (new numerator) and the denominator (7) stays the same.

$$
\begin{array}{r}
3 \\
7\,\overline{)\,23} \\
\underline{21} \\
2 \\
= 3\frac{2}{7}
\end{array}
$$

Example 6 Write $\frac{18}{11}$ as a mixed number.

Example 6 is done the same way. 11 goes into 18 one time, $(18 \div 11 = 1)$, with 7 left over (remainder). Therefore 1 is the whole number (out front) with 7 becoming the new numerator and 11 remaining as the denominator.

$$\begin{array}{r} 1 \\ 11\,\overline{)\,18} \\ \underline{11} \\ 7 \end{array}$$

$$= 1\frac{7}{11}$$

Adding Fractions

When adding fractions, we must pay attention to one important detail: the denominators of the fractions we are adding. This will tell us if the addition will be simple or more difficult.

To find the sum of two fractions with the same denominator, add the numerators and keep the denominator unchanged in the answer. In Example 7, the numerators can be added to find the numerator of the answer since the denominators for the two fractions in the question are equivalent. The numerator of the sum will be $3+2=5$, and the denominator of the sum will be unchanged.

Example 7 Simplify $\frac{3}{7}+\frac{2}{7}$.

$$\begin{aligned} \frac{3}{7}+\frac{2}{7} &= \frac{3+2}{7} \\ &= \frac{5}{7} \end{aligned}$$

Example 8 (below) is solved in same manner, with the numerator of the answer equal to the sum of the original numerators. Since the answer is an improper fraction, *it may be necessary to convert to a mixed number, as shown previously.*

Example 8 Simplify $\frac{5}{9} + \frac{6}{9}$.

$$\begin{aligned}\frac{5}{9} + \frac{6}{9} &= \frac{5+6}{9} \\ &= \frac{11}{9} \\ &= 1\frac{2}{11}\end{aligned}$$

Adding Fractions with different denominators

To find the sum of two fractions that have different denominators, find a common multiple of the denominators. A **common multiple** is a specific number that is a multiple of two or more numbers. For instance, 24 is a common multiple of the numbers 4 and 8 because $4 \cdot 6$ is 24 and $8 \cdot 3$ is also 24.

Another way to find a common denominator is by multiplying the two denominators together. This product will be a common multiple of the two (or more) individual denominators, as shown in Step 1 of the general example below. If you choose this method, you will probably have to simplify your fraction to lowest terms as shown in Example 1 and Example 2.

$$\begin{aligned}\frac{a}{b} + \frac{c}{d} &= \frac{?}{b \cdot d} + \frac{?}{d \cdot b} && \text{Step 1} \\ &= \frac{a \cdot d}{b \cdot d} + \frac{c \cdot b}{d \cdot b} && \text{Step 2} \\ &= \frac{(a \cdot d) + (c \cdot b)}{b \cdot d} && \text{Step 3}\end{aligned}$$

After step one, return to the fraction addition problem and multiply each denominator by the appropriate value to obtain a common denominator (the common multiple of the denominators), and also multiply the respective numerators by the same values (that the denominator was multiplied by) to obtain the fractions to be added, as shown is Step 2 above. Then, follow the steps to add fractions with the same denominator, seen above in Step 3.
In Example 9 below, the denominators (6 and 9) are multiplied together to find the common denominator to be used, which is $6 \cdot 9 = \mathbf{54}$. The numerator 1 will also be multiplied by 9 to maintain equality, resulting in the fraction $\frac{9}{54}$.

The numerator 2 of the second fraction will also be multiplied by 6, resulting in the fraction $\frac{12}{54}$. These two fractions, with common denominators, can be added according to the method described for Examples 7 and 8 from above. Notice that the answer $\left(\frac{21}{54}\right)$ will need to be simplified using a common factor of 3.

Example 9 What is $\dfrac{1}{6} + \dfrac{2}{9} =$

$$\begin{aligned}
\frac{1}{6} + \frac{2}{9} &= \frac{1 \cdot 9}{6 \cdot 9} + \frac{2 \cdot 6}{9 \cdot 6} \\
&= \frac{9}{54} + \frac{12}{54} \\
&= \frac{9 + 12}{54} \\
&= \frac{21}{54} \\
&= \frac{7 \cdot 3}{18 \cdot 3} \\
&= \frac{7}{18}
\end{aligned}$$

Example 10 demonstrates a method of finding the sum of mixed numbers.

Example 10 Find the sum of $2\dfrac{3}{4} + 1\dfrac{3}{7}$.

We have:

$$\begin{aligned}
2\frac{3}{4} + 1\frac{3}{7} &= \frac{11}{4} + \frac{10}{7} \\
&= \left(\frac{11 \cdot 7}{4 \cdot 7} + \frac{10 \cdot 4}{7 \cdot 4}\right) \\
&= \frac{117}{28} \\
&= 4\frac{5}{28}
\end{aligned}$$

Finding the Least Common Denominator (LCD)

Example 11 Find the least common denominator of $\frac{3}{14}$ and $\frac{5}{32}$.

Step 1: Find the prime factorization of each denominator in the (addition or subtraction problem). Prime numbers are those numbers that only have factors of 1 and the number itself, such as 2,3,5,7 and 11 (not 9). Factor each denominator into primes as in the example below, first considering the smallest prime number (2) and trying larger primes if the smaller are not factors of the given denominator. So, let's find the prime factors of 14 and 32.

$$14 = 2 \cdot 7$$
$$32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

Once the prime factorization of each denominator is complete, find the instances that each prime factor appears in the factorization of each denominator, as shown in Step 2. Note the greatest number of times that each prime number appears for individual denominators.

Step 2:

for 14 →"2" appears once and "7" appears once

for 32 → "2" appears five times

Listing each prime number in the greatest number of appearances and finding the product of these prime numbers will produce the least common denominator, as shown in Step 3.

Step 3 Since "2" appears only once in the factorization of "14" and appears five times in the factorization of "32", "2" will be listed five times; "7" appears once for "14". So, we have 2,2,2,2,2 and 7, all of which will be multiplied together to find the least common denominator. Therefore $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 7 = 224$ is the least common denominator of 14 and 32.

Multiplying Fractions

To find the product of fractions, multiply straight across the numerators to obtain the numerator of the answer, and multiply straight across the denominators to find the new denominator, as in Examples 1 and 2 below.

Example 12 Simplify $\frac{3}{7} \cdot \frac{2}{5}$.

$$\begin{aligned}\frac{3}{7} \cdot \frac{2}{5} &= \frac{3 \cdot 2}{7 \cdot 5} \\ &= \frac{6}{35}\end{aligned}$$

Example 13 Simplify $\frac{2}{5} \cdot \frac{-2}{9}$.

$$\begin{aligned}\frac{2}{5} \cdot \frac{-2}{9} &= \frac{2 \cdot (-2)}{5 \cdot 9} \\ &= \frac{-4}{45} \\ &= -\frac{4}{45}\end{aligned}$$

If there is a mixed number in the question, convert it to an improper fraction before finding the product, as in Example 14.

Example 14 Find the product of $2\frac{3}{4}$ and $\frac{4}{7}$.

$$\begin{aligned}2\frac{3}{4} \cdot \frac{4}{7} &= \frac{11}{\cancel{4}} \cdot \frac{\cancel{4}}{7} \\ &= \frac{11}{7} \\ &= 1\frac{4}{7}\end{aligned}$$

Dividing Fractions

To find the quotient of two fractions, take the reciprocal (flip the fraction) of the divisor (the second fraction) and multiply the two fractions, as shown in Examples 15 and 16 below.

Example 15 $\frac{3}{7} \div \frac{2}{5}$.

To divide these fractions, we must change the operation to multiplication. To do this, we take the reciprocal of the second fraction $\frac{5}{2}$ which is $\frac{2}{5}$. Then we can change the operation to multiplication. We can now follow our rules of multiplication to simplify the problem.

$$\begin{aligned}\frac{3}{7} \div \frac{2}{5} &= \frac{3}{7} \cdot \frac{5}{2} \\ &= \frac{15}{14} \\ &= 1\frac{1}{14}\end{aligned}$$

Example 16 $\dfrac{2}{5} \div \dfrac{1}{9}$.

$$\begin{aligned}\frac{2}{5} \div \frac{1}{9} &= \frac{2}{5} \cdot \frac{9}{1} \\ &= \frac{18}{5} \\ &= 3\frac{3}{5}\end{aligned}$$

Comparing Fractions

To find the fraction with greatest value, cross-multiply, as shown below. The larger fraction will correspond to the larger product obtained from the cross-multiplication.

Example 17 Which fraction is larger $\dfrac{5}{8}$ or $\dfrac{4}{7}$?
First, we cross multiply the numerator of the left fraction with the denominator of the right fraction. The product is $5 \cdot 7 = 35$. Write this value above the left fraction. Next, multiply the numerator of the right fraction by the denominator of the left which gives us $4 \cdot 8 = 32$. Write this value above the right fraction. The number above the fraction on the left is larger, therefore, $\frac{5}{8}$ is larger.

$$\overbrace{\frac{5}{8}}^{5\cdot 7=35} > \overbrace{\frac{4}{7}}^{4\cdot 8=32}$$

Practice Problems

1. Find the least common denominator of each pair of fractions.

   (a) $\frac{7}{9}$ and $\frac{1}{6}$ (b) $\frac{3}{11}$ and $\frac{5}{8}$

2. Add the fractions and write the answer in simplest form (no improper fractions).

   3. $\frac{7}{9} + \frac{1}{6}$ 4. $2\frac{1}{9} + 3\frac{4}{5}$

3. Subtract the fractions and write the answer in simplest form (no improper fractions).

   5. $\frac{4}{9} - \frac{1}{3}$ 6. $3\frac{2}{7} - 2\frac{1}{13}$

4. Multiply the fractions and write the answer in simplest form (no improper fractions).

   7. $\frac{1}{4} \cdot \frac{3}{8}$ 8. $2\frac{1}{3} \cdot 1\frac{2}{5}$

5. Divide the fractions and write the answer in simplest form (no improper fractions).

   9. $\frac{1}{4} \div \frac{1}{5}$ 10. $2\frac{3}{8} \div \frac{3}{5}$

Solutions to Practice Problems

1. **18**. We take the factors of each denominator to get the LCD (least common denominator). If any factors repeat, we take the factor that appears most often. We know that $9 = 1 \cdot 3 \cdot 3 = 3^2$ and $6 = 1 \cdot 2 \cdot 3$. Notice how both denominators (6 and 9) have factors of 3, however, 9 has two factors of 3 $(3 \cdot 3)$, so we use this factor for our LCD. We also see that 2 is a factor of 6, but not of 9, so 2 also belongs in the LCD. Therefore the LCD$=3 \cdot 3 \cdot 2 = 18$.

2. **88** Re-writing each denominators in terms of their factors gives us: $11 = 1 \cdot 11$ and $8 = 1 \cdot 2 \cdot 4$. We take the factors that appear in each denominator. There are no repeated factors (other than 1).Therefore, the LCD$=11 \cdot 2 \cdot 4 = 88$.

3. $\mathbf{\frac{17}{18}}$ We must first find a common denominator which is 18. Then, we multiply the first fraction $\frac{7}{9}$ by $\frac{2}{2}$, and the second fraction, $\frac{1}{6}$ by $\frac{3}{3}$. Thus,
$$\begin{aligned}
\frac{7}{9} + \frac{1}{6} &= \frac{7 \cdot 2}{9 \cdot 2} + \frac{1 \cdot 3}{6 \cdot 3} \\
&= \frac{14}{18} + \frac{3}{18} \\
&= \frac{14 + 3}{18} \\
&= \frac{17}{18}
\end{aligned}$$

4. $\mathbf{5\frac{41}{45}}$.We are adding two mixed numbers, therefore add the whole parts $3+2 = 5$. Add the fractions separately. The denominators are both factors of 45. This is the smallest number that 5 and 9 go into. Therefore 45 is the LCD. Simplifying we get:
$$\begin{aligned}
2\frac{1}{9} + 3\frac{4}{5} &= 2\frac{1 \cdot 5}{9 \cdot 5} + 3\frac{4 \cdot 9}{5 \cdot 9} \\
&= 2\frac{5}{45} + 3\frac{36}{45} \\
&= 2 + 3 + \frac{5 + 36}{45} \\
&= 5\frac{41}{45}
\end{aligned}$$

5. $\mathbf{\frac{1}{9}}$. We see that 3 is a factor of 9; therefore, the LCD is 9. We only have to multiply the second fraction by $\frac{3}{3}$ since the first fraction already has a denominator of 9.

$$\begin{aligned}\frac{4}{9}-\frac{1}{3}&=\frac{4}{9}-\frac{1\cdot 3}{3\cdot 3}\\&=\frac{4}{9}-\frac{3}{9}\\&=\frac{1}{9}\end{aligned}$$

6. $\mathbf{1\frac{19}{91}}$.First change each mixed number to an improper fraction. This yields $\dfrac{23}{7}$ and $\dfrac{27}{13}$. Seven and 13 are both prime numbers therefore, the LCD is $7\cdot 13 = 91$. Thus, the first fraction will be multiplied by $\frac{13}{13}$, and the second fraction by $\frac{7}{7}$. This yields,

$$\begin{aligned}\frac{23}{7}-\frac{27}{13}&=\frac{23\cdot 13}{7\cdot 13}-\frac{27\cdot 7}{13\cdot 7}\\&=\frac{299}{91}-\frac{189}{91}\\&=\frac{110}{91}\\&=1\frac{19}{91}\end{aligned}$$

7. $\mathbf{\frac{3}{32}}$. When multiplying fractions we can simplify the fractions first (if possible) or we can just multiply straight across. To be consistent, we will multiply straight across. Therefore,

$$\begin{aligned}\frac{1}{4}\cdot\frac{3}{8}&=\frac{1\cdot 3}{4\cdot 8}\\&=\frac{3}{32}\end{aligned}$$

8. $\mathbf{3\frac{3}{15}}$. First we will change the mixed numbers to improper fractions. We will then multiply the fractions straight across and simplify if necessary.

$$\begin{aligned} 2\frac{1}{3} \cdot 1\frac{2}{5} &= \frac{7}{3} \cdot \frac{7}{5} \\ &= \frac{7 \cdot 7}{3 \cdot 5} \\ &= \frac{49}{15} \\ &= 3\frac{4}{15} \end{aligned}$$

9. $\mathbf{1\frac{1}{4}}$To divide these fractions, take the reciprocal (flip) so the second fraction and take the product of the two fractions.

$$\begin{aligned} \frac{1}{4} \div \frac{1}{5} &= \frac{1}{4} \cdot \frac{5}{1} \\ &= \frac{1 \cdot 5}{4 \cdot 1} \\ &= \frac{5}{4} \\ &= 1\frac{1}{4} \end{aligned}$$

10. $\mathbf{3\frac{23}{24}}$ First change our mixed number into an improper fraction. Take the reciprocal of the second fraction and multiply the fractions together.

$$\begin{aligned} 2\frac{3}{8} \div \frac{3}{5} &= \frac{19}{8} \cdot \frac{5}{3} \\ &= \frac{19 \cdot 5}{8 \cdot 3} \\ &= \frac{95}{24} \\ &= 3\frac{23}{24} \end{aligned}$$