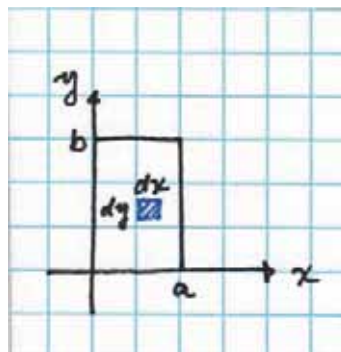


## Examples

### 1. Area of a Rectangle

$$A = \int dA = \int_0^b \int_0^a dx \cdot dy = \int_0^b x \Big|_0^a \cdot dy$$

$$A = \int_0^b a \cdot dy = a \cdot y \Big|_0^b = a \cdot b$$

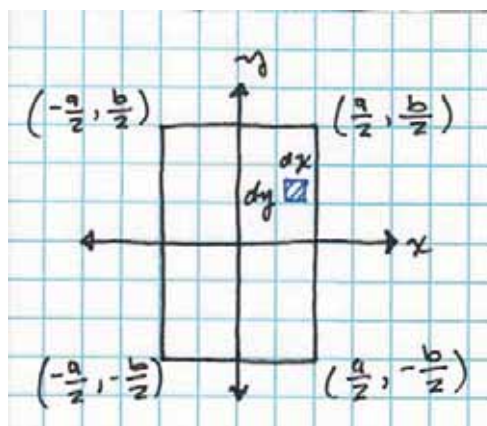


### 2. Area of a Rectangle

$$A = \int dA = \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} dx \cdot dy = \int_{-b/2}^{b/2} x \Big|_{-a/2}^{a/2} \cdot dy$$

$$A = \int_{-b/2}^{b/2} \left[ \frac{a}{2} - \left( -\frac{a}{2} \right) \right] \cdot dy = \int_{-b/2}^{b/2} a \cdot dy$$

$$A = a \cdot [y]_{-b/2}^{b/2} = a \cdot \left[ \frac{b}{2} - \left( -\frac{b}{2} \right) \right] = a \cdot b$$



### 3. Area of a Triangle

$$A = \int dA = \int_0^b c \cdot dy$$

$$c = a_1 + a_2 \cdot y$$

$$c = a \quad @ \quad y = 0 \quad \rightarrow \quad a_1 = a$$

$$c = 0 \quad @ \quad y = b$$

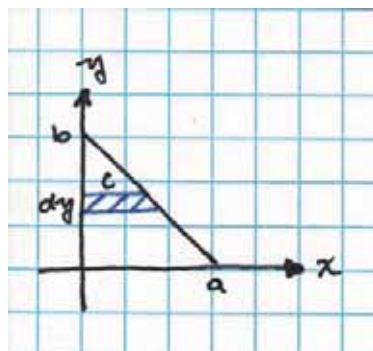
$$c = a_1 + a_2 \cdot y = a + a_2 \cdot b = 0 \quad \text{thus} \quad a_2 = -a/b$$

$$c = a + \left( -\frac{a}{b} \right) \cdot y = a - \frac{a}{b} \cdot y$$

$$A = \int_0^b c \cdot dy = \int_0^b \left[ a - \frac{a}{b} \cdot y \right] \cdot dy = \int_0^b a \cdot dy - \frac{a}{b} \int_0^b y \cdot dy$$

$$A = a \cdot y \Big|_0^b - \frac{a}{b} \cdot \frac{y^2}{2} \Big|_0^b = a \cdot b - \frac{a}{b} \cdot \frac{b^2}{2} = a \cdot b - \frac{1}{2} \cdot a \cdot b$$

$$A = \frac{1}{2} a \cdot b$$

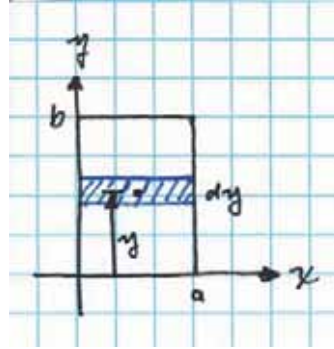


4. First Moment of Area – Rectangle

$$Q_x = \int y \cdot dA$$

$$Q_x = \int_a^b a \cdot y \cdot dy = a \cdot \frac{y^2}{2} \Big|_0^b$$

$$Q_x = \frac{a \cdot b^2}{2}$$

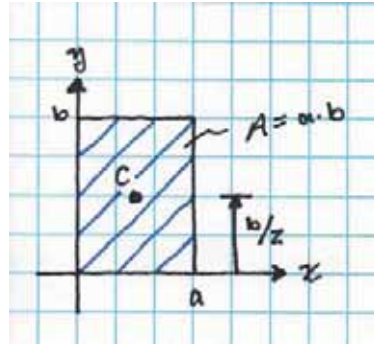


5. First Moment of Area – Rectangle

$$Q_x = \bar{y} \cdot A$$

$$Q_x = \left(\frac{b}{2}\right) (a \cdot b)$$

$$Q_x = \frac{a \cdot b^2}{2}$$



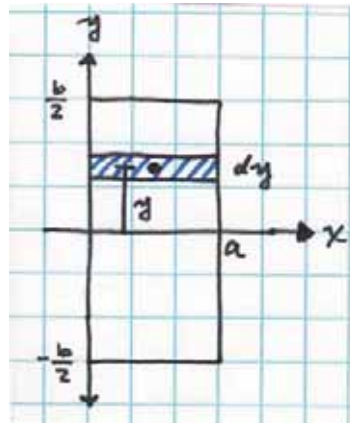
6. First Moment of Area – Rectangle

$$Q_x = \int y \cdot dA$$

$$Q_x = \int_{-b/2}^{b/2} a \cdot y \cdot dy = a \cdot \frac{y^2}{2} \Big|_{-b/2}^{b/2}$$

$$Q_x = a \cdot \left[ \frac{b^2}{8} - \frac{1}{2} \left( -\frac{b}{2} \right)^2 \right] = a \cdot \left[ \frac{b^2}{8} - \frac{b^2}{8} \right]$$

$$Q_x = 0$$



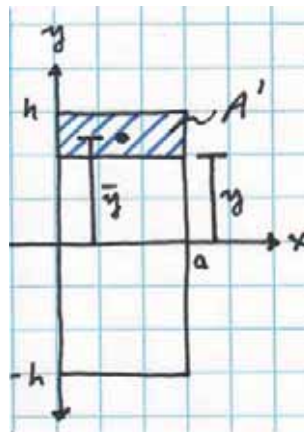
7. First Moment of Area – Rectangle (A')

$$A' = a \cdot (h - y)$$

$$\bar{y} = \frac{h + y}{2}$$

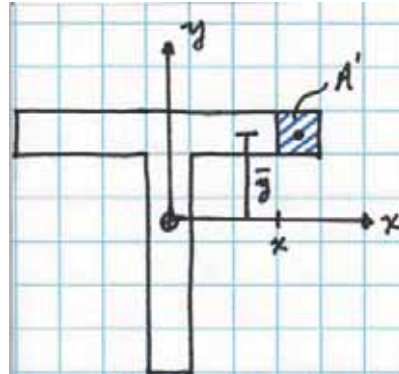
$$Q_x = \bar{y} \cdot A' = \left( \frac{h + y}{2} \right) \cdot a \cdot (h - y)$$

$$Q_x = \frac{a}{2} \cdot (h^2 - y^2)$$



8. First Moment of Area – T-Section (A')

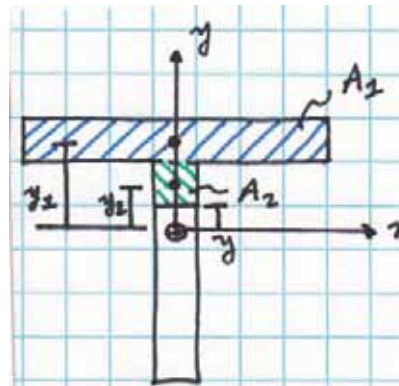
$$Q_x = A' \cdot \bar{y} \quad @ \text{ value of } x$$



9. First Moment of Area – T-Section (A')

$$Q_x = \sum \bar{y}_i \cdot A'_i$$

$$Q_x = \bar{y}_1 \cdot A_1 + \bar{y}_2 \cdot A_2 \quad @ \text{ value of } y$$



10. Moment of Inertia

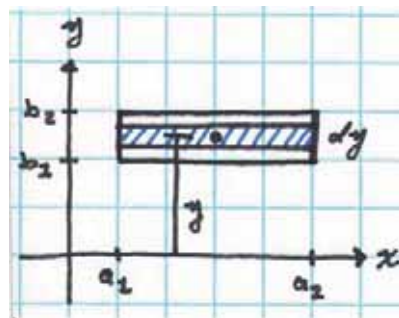
$$I_x = \int y^2 \cdot dA$$

$$I_x = \int_{b_1}^{b_2} y^2 \cdot (a_2 - a_1) \cdot dy$$

$$I_x = (a_2 - a_1) \int_{b_1}^{b_2} y^2 \cdot dy$$

$$I_x = (a_2 - a_1) \left( \frac{y^3}{3} \right)_{b_1}^{b_2} = (a_2 - a_1) \left[ \frac{b_2^3}{3} - \frac{b_1^3}{3} \right]$$

$$I_x = \left( \frac{a_2 - a_1}{3} \right) (b_2^3 - b_1^3)$$



11. Moment of Inertia  $\rightarrow b_1 = -b_2$

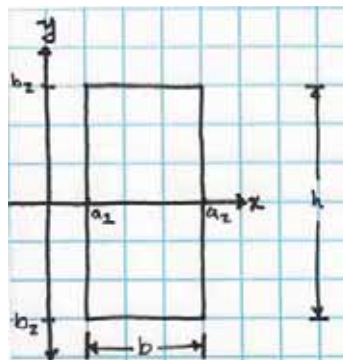
$$I_x = \left( \frac{a_2 - a_1}{3} \right) (b_2^3 + b_2^3)$$

$$I_x = \left( \frac{a_2 - a_1}{3} \right) 2b_2^3$$

$$I_x = \frac{2}{3} (a_2 - a_1) b_2^3$$

$$I_x = \frac{2}{3} b \cdot \left( \frac{h^3}{8} \right)$$

$$I_x = \frac{b h^3}{12}$$

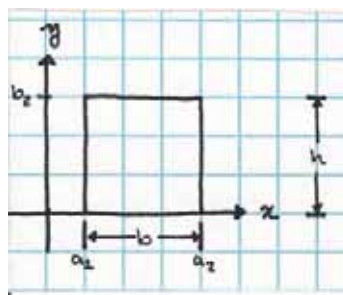


12. Moment of Inertia  $\rightarrow b_1 = 0$

$$I_x = \left( \frac{a_2 - a_1}{3} \right) (b_2^3 - b_1^3)$$

$$I_x = \left( \frac{a_2 - a_1}{3} \right) b_2^3$$

$$I_x = \frac{b \cdot h^3}{3}$$



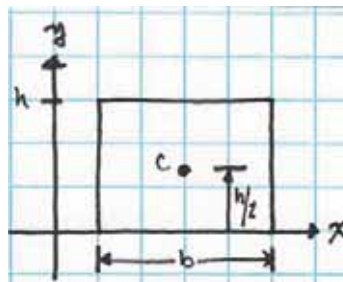
13. Parallel Axis Theorem – Moment of Inertia

$$I_x = I_{x_c} + A \cdot d^2$$

$$I_x = \frac{b \cdot h^3}{12} + b \cdot h \cdot \left( \frac{h}{2} \right)^2$$

$$I_x = \frac{b \cdot h^3}{12} + \frac{b \cdot h^3}{4}$$

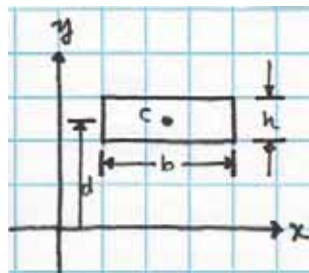
$$I_x = \frac{b \cdot h^3}{3}$$



14. Parallel Axis Theorem – Moment of Inertia

$$I_x = I_{x_c} + A \cdot d^2$$

$$I_x = \frac{b \cdot h^3}{12} + (b \cdot h) d^2$$



### 15. Moment of Inertia – Triangle

$$I_x = \int y^2 \cdot dA$$

$$I_x = \int_0^h 2y^2 \cdot s \cdot dy$$

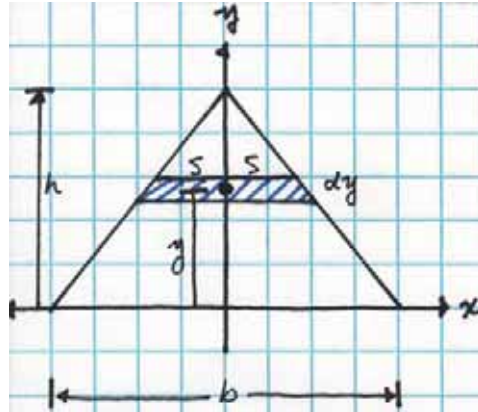
$$I_x = 2 \int_0^h y^2 \cdot \left[ \frac{b}{2h} \cdot (h - y) \right] \cdot dy$$

$$I_x = \int_0^h \frac{b}{h} \cdot (h \cdot y^2 - y^3) \cdot dy$$

$$I_x = \frac{b}{h} \cdot \left[ h \cdot \frac{y^3}{3} \Big|_0^h - \frac{y^4}{4} \Big|_0^h \right]$$

$$I_x = \frac{b}{h} \cdot \left[ \frac{h^4}{3} - \frac{h^4}{4} \right] = \frac{b}{h} \cdot \left[ \frac{4h^4}{12} - \frac{3h^4}{12} \right]$$

$$I_x = \frac{b \cdot h^4}{12h} = \frac{b \cdot h^3}{12}$$



### 16. Product of Inertia

$$I_{xy} = \int x \cdot y \cdot dA$$

$$I_{xy} = \int_{b_1}^{b_2} \int_{a_1}^{a_2} x \cdot y \cdot dx \cdot dy$$

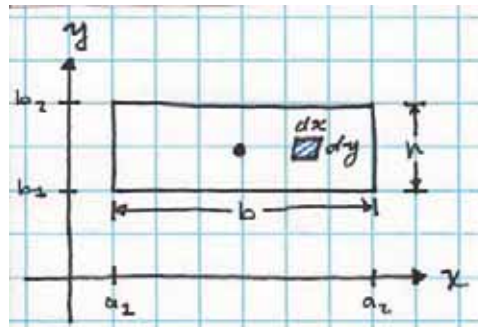
$$I_{xy} = \int_{b_1}^{b_2} \left[ \int_{a_1}^{a_2} x \cdot dx \right] \cdot y \cdot dy$$

$$I_{xy} = \int_{b_1}^{b_2} \frac{x^2}{2} \Big|_{a_1}^{a_2} \cdot y \cdot dy = \int_{b_1}^{b_2} \left( \frac{a_2^2 - a_1^2}{2} \right) \cdot y \cdot dy$$

$$I_{xy} = \left( \frac{a_2^2 - a_1^2}{2} \right) \left( \frac{b_2^2 - b_1^2}{2} \right) = (a_2 - a_1) \left( \frac{a_2 + a_1}{2} \right) (b_2 - b_1) \left( \frac{b_2 + b_1}{2} \right)$$

$$I_{xy} = b \cdot dx \cdot h \cdot dy = dx \cdot dy \cdot (b \cdot h)$$

$$I_{xy} = dx \cdot dy \cdot A$$

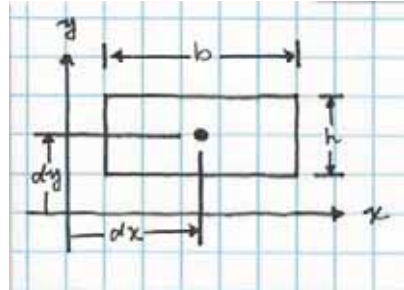


### 17. Parallel Axis Theorum – Product of Inertia

$$I_{xy} = I_{xy_c} + dx \cdot dy \cdot A$$

$I_{xy_c} = 0$  if the section has at least one plane of symmetry.

$I_{xy}$  can be positive, negative or zero.



### 18. Polar Moment of Inertia – Rircular Rod

$$dA = r \cdot d\theta \cdot dr$$

$$J = \int r^2 \cdot dA$$

$$J = \int_0^{2\pi} \int_0^R r^2 \cdot (r \cdot d\theta \cdot dr) = \int_0^{2\pi} d\theta \int_0^R r^3 \cdot dr$$

$$J = \theta \Big|_0^{2\pi} \cdot \left( \frac{r^4}{4} \right)_0^R = 2\pi \left( \frac{R^4}{4} \right)$$

$$J = \frac{\pi \cdot R^4}{2}$$

