QUESTION SIX (12 marks) Use a separate writing booklet. **Marks**

(a) Prove by mathematical induction that for all positive integer values of n, **3**

$$\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ldots+\frac{n}{(n+1)!}=1-\frac{1}{(n+1)!}.$$

(b) A film director must decide whether a stuntman is able to perform a dangerous stunt. The stuntman must leap from a building onto the centre of some erected scaffolding. The centre of the scaffolding is 5 m below his initial position and at a horizontal distance of 14 m. The stuntman jumps at an angle of 30° above the horizontal. Let the stuntman's initial velocity be V, and let x and y be his horizontal and vertical displacements respectively from his initial position. You may assume that the velocity and displacement equations are:

$$\dot{x}=V\cos 30^\circ \qquad \dot{y}=-10t+V\sin 30^\circ$$

$$x=Vt\cos 30^\circ \qquad y=-5t^2+Vt\sin 30^\circ$$

(i) Show that the Cartesian equation of the stuntman's path is **2**

$$y=-\frac{20x^2}{3V^2}+\frac{x}{\sqrt{3}}.$$

(ii) Hence determine the required initial velocity V so that he lands in the centre of the scaffolding. Write your answer to the nearest m/s. **2**

(iii) Safety requirements are such that if the impact velocity is greater than 15 m/s, then padding must be placed on the scaffolding. Assuming that the stuntman leaps at the required speed, determine whether or not padding is needed. **2**

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