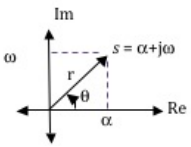# THE LAPLACE TRANSFORMATION

REVIEW OF COMPLEX NUMBERS

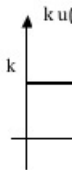A complex number can be represented in rectangular or polar form.

Rectangular		Polar
$s = \alpha + j\omega$		$s = r\angle\theta = re^{j\theta}$
$\alpha = \text{Re}(s)$ (real part)		$r = |s| = (\omega^2 + \alpha^2)^{1/2}$ (mag)
$\omega = \text{Im}(s)$ (imag part)		$\theta = tan^{-1}(\omega/\alpha)$ (phase)

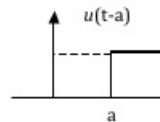## USEFUL FUNCTIONS AND THEIR LAPLACE TRANSFORM

1. Step function:

$$ku(t) \quad = \quad \begin{cases} k & t \geq 0 \\ 0 & otherwise \end{cases}$$

A delayed (shifted) unit step function with delay value $a > 0$.

$$u(t-a) \quad = \quad \begin{cases} 1 & t-a \geq 0 \, or \, t \geq a \\ 0 & t < a \end{cases}$$
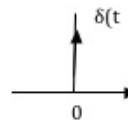
2. Impulse Function.

Definition. The impulse signal has an infinite amplitude and zero duration.

Graphically, $\delta(t)$ is represendted by an arrow at $t = 0$

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$

3. Ramp function.

$$f(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$