

THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS

Mathematics 031

Test 2
90 minutes

November 1, 2007

PART A (18 marks)

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1. Consider the following systems of equations.

$$\begin{array}{lll}
 (1) & x + 13y^2 - z = 5 & (2) \quad \frac{x}{2} - \frac{y}{3} + \frac{z}{4} = 5 \quad (3) \quad \frac{x}{2} - \frac{y}{3} + \frac{z}{4} = \sqrt{5} \\
 & \frac{2}{x} - \frac{3}{y} + \frac{4}{z} = 5 & x(3y + 4z) = 1 \quad x + 13^2y - \sqrt{13}z = \frac{1}{5} \\
 & \frac{x}{2} - \frac{y}{3} + \frac{z}{4} = \sqrt{5} & x + 13y^2 - z = 5 \quad x - (3y + 4z) = 1
 \end{array}$$

Which of the systems of equations is linear in x, y and z ?

A: all of them	B: (1) only	C: (2) only	D: (3) only	E: none of them
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2. Find the augmented matrix for the following system of linear equations.

$$\begin{array}{l}
 x = 2y - 1 \\
 3y = 4x \\
 0 = y + 3
 \end{array}$$

A: $\left[\begin{array}{cc c} 1 & -2 & -1 \\ 4 & 3 & 0 \\ 0 & 1 & 3 \end{array} \right]$	B: $\left[\begin{array}{cc c} 1 & 2 & -1 \\ 3 & 0 & 4 \\ 0 & -1 & 3 \end{array} \right]$	C: $\left[\begin{array}{cc c} 1 & -2 & -1 \\ -4 & 3 & 0 \\ 0 & -1 & 3 \end{array} \right]$
D: $\left[\begin{array}{cc c} 1 & -2 & -1 \\ 0 & 3 & 4 \\ -1 & 0 & 3 \end{array} \right]$	E: $\left[\begin{array}{cc c} 1 & 2 & -1 \\ 3 & 4 & 0 \\ 0 & -1 & 3 \end{array} \right]$	

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3. Which one of the following matrices is **not** in row-reduced echelon form?

A: $\left[\begin{array}{ccc} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{array} \right]$	B: $\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$	C: $\left[\begin{array}{ccc} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{array} \right]$
D: $\left[\begin{array}{ccc} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{array} \right]$	E: $\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right]$	

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4. Let $A = \begin{bmatrix} 1 & 3 & -1 \\ -1 & -2 & 3 \\ 2 & 5 & -4 \end{bmatrix}$ and let B be the row-reduced echelon form of A . Then the first row of B is

A: $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$	B: $\begin{bmatrix} 1 & 0 & -7 \end{bmatrix}$	C: $\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$	D: $\begin{bmatrix} 1 & 0 & 7 \end{bmatrix}$	E: $\begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$
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5. The system of linear equations

$$\begin{aligned} x_1 - 2x_2 - 3x_3 + 4x_4 &= 1 \\ -x_1 + x_2 + 3x_3 - 4x_4 &= -2 \\ -2x_1 + 4x_2 + 6x_3 - 8x_4 &= -2 \end{aligned}$$

has

A: no solutions	B: exactly one solution
C: a one-parameter family of solutions	D: a two-parameter family of solutions
E: a three-parameter family of solutions	

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6. The system of linear equations

$$\begin{aligned} x_1 - 2x_2 - 3x_3 + 4x_4 &= 1 \\ -x_1 + x_2 + 3x_3 - 4x_4 &= 2 \\ -2x_1 + 4x_2 + 6x_3 - 8x_4 &= -1 \end{aligned}$$

has

A: no solutions	B: exactly one solution
C: a one-parameter family of solutions	D: a two-parameter family of solutions
E: a three-parameter family of solutions	

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7. Find the value(s) of k for which the system of linear equations with augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & k+4 & 2k+8 \end{array} \right] \text{ has no solution.}$$

A: $k = 4$ only	B: $k \neq 4$	C: $k = -4$ only	D: $k \neq -4$	E: no value of k
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8. Find the value(s) of k for which the system of linear equations with augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & k+4 & 2k+8 \end{array} \right] \text{ has infinitely many solutions.}$$

A: $k = 4$ only	B: $k \neq 4$	C: $k = -4$ only	D: $k \neq -4$	E: no value of k
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9. Find the value(s) of k for which the system of linear equations with augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & k+4 & 2k+8 \end{array} \right] \text{ has exactly one solution.}$$

A: $k = 4$ only	B: $k \neq 4$	C: $k = -4$ only	D: $k \neq -4$	E: no value of k
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10. Let A be a 2×3 matrix, B a 3×3 matrix and C a 4×2 matrix. Then CAB is

A: a 2×3 matrix.	B: a 3×3 matrix	C: a 2×4 matrix
D: a 3×2 matrix	E: a 4×3 matrix	

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11. If A is a 2×3 matrix and B a 3×3 matrix, then which one of the following is **not** defined?

A: AA^T	B: AB	C: BA^T	D: $B + A^T A$	E: $B + AA^T$
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12. If $5 \begin{bmatrix} 9 & -2 \\ 0 & -20 \end{bmatrix} - 2 \begin{bmatrix} 7 & -5 \\ 20 & 0 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $b =$

A: -7	B: 20	C: 9	D: 0	E: -20
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13. If $AB = C = [c_{ij}]$ where $A = \begin{bmatrix} 3 & 0 & 14 \\ -14 & -1 & 1 \\ 2 & 2 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 & 42 \\ 4 & 0 & 14 \\ -1 & 2 & -14 \end{bmatrix}$ then c_{32} is

A: 0	B: -14	C: 14	D: 3	E: -1
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14. Let $A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$. Find A^{15} .

A: $\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$	B: $\begin{bmatrix} 1 & 0 \\ -1 & 15 \end{bmatrix}$	C: $\begin{bmatrix} 15 & 0 \\ -1 & 1 \end{bmatrix}$	D: $\begin{bmatrix} 1 & 0 \\ -15 & 1 \end{bmatrix}$	E: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
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15. Let $A = \begin{bmatrix} 12 & 4 \\ 2 & 1 \end{bmatrix}$. If $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then the value of d is

A: $-\frac{1}{2}$	B: 3	C: -1	D: $\frac{1}{2}$	E: 1
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16. The matrix $A = \begin{bmatrix} -1 & 3 & -1 \\ 1 & -2 & 4 \\ 2 & -5 & k \end{bmatrix}$ has no inverse when $k =$

A: -5	B: -3	C: 3	D: 5	E: 2
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17. The matrix $A = \begin{bmatrix} 2 & 3 \\ 4 & k+1 \end{bmatrix}$ has no transpose when $k =$

A: 5	B: 0	C: -5	D: 1	E: no value of k
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18. If $A^{-1} = \begin{bmatrix} 2 & 1 & -7 \\ -7 & -4 & 2 \\ 21 & 3 & 3 \end{bmatrix}$, where $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, then in the solution of the system of linear equations

$$\begin{aligned} ax + by + cz &= 1 \\ dx + ey + fz &= -2 \\ gx + hy + iz &= 3 \end{aligned}$$

the value of y is

A: -7	B: 21	C: -4	D: 7	E: 2
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19. Consider the following system of linear equations in x, y and z .

$$\begin{array}{rcrcrcrcrcl} x & + & 2y & + & 3z & = & 1 \\ 2x & + & 4y & + & 3z & = & 8 \\ 3x & + & 6y & + & 9z & = & 3 \end{array}$$

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- (a) What is the augmented matrix for this system of equations?

Answer: _____

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- (b) What is the row-reduced echelon form of this augmented matrix?

Answer: _____

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- (c) What is the solution for this system of equations?

$x =$ _____

$y =$ _____

$z =$ _____

20. In each of the following, for the given augmented matrix, find the solution to the corresponding system of linear equations.

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(a) $\left[\begin{array}{ccc|c} 0 & 1 & -4 & 11 \\ 0 & -3 & 0 & -9 \\ 2 & 0 & 0 & -2 \end{array} \right]$

$x =$ _____

$y =$ _____

$z =$ _____

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(b) $\left[\begin{array}{ccc|c} 1 & 3 & 2 & -4 \\ -1 & 3 & -2 & -2 \end{array} \right]$

$x =$ _____

$y =$ _____

$z =$ _____

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21. Using the method of row reduction, find A^{-1} where $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & -2 \\ 3 & 5 & -2 \end{bmatrix}$.

Answer: _____